Reg. No. : ....

Name : ....

Fourth Semester B.Tech. Degree Examination, February 2015
(2008 Scheme)
08.401 : ENGINEERING MATHEMATICS – III (CMPUNERFHB)
(Special Supplementary)

Time : 3 Hours Max. Marks : 100

Answer **all** questions from Part **A** and **one full** question from **each** Module of Part **B**.

PART – A

1. Show that Suiz is analytic every where in the complex plane and find its derivative.
2. Examine whether $xy^2 + x^2y$ can be the real part of an analytic function.
3. If a function is analytic show that it is independent of $\bar{z}$.
4. Find the image of the half plane x < c under the mapping w = 1/z.
5. Using Cauchy's integral formula evaluate $\int_c \frac{dz}{z^2+4}$ where c is the circle $|z-i| = 2$.
6. Expand $\frac{z-1}{z^2}$ as a Taylar series in powers of z – 1 and state the region of convergence.
7. Find the poles and residues of $\frac{\text{suiz}}{z^4}$.
8. Solve the equations by Gauss – Elimination method

   $2x + 3y - z = 5$

   $4x + 4y - 3z = 3$

   $2x - 3y + 2z = 2$

9. Fit a polynomial to the data

x :	0	1	3	4
y :	–12	0	6	12

10. Use Trapezoidal rule to evaluate $\int_0^1 \frac{1}{1+x^2}dx$ with h = 0.5 and hence deduce an approximate value of π. **(10×4=40 Marks)**

PART – B

Module – I

11. a) Prove that the function $F(z) = \frac{x^3(1+i) - y^3(1-i)}{x^2+y^2}$ when $z \neq 0$, $F(z) = 0$. When $z = 0$ is not analytic at $z = 0$ even though Cauchy-Riemann equations are satisfied at that point.

b) Construct the analytic function whose real part is

$\sin x \cosh y + 2 \cos x \sinh y + x^2 - y^2 + 4xy$.

c) Find the bilinear transformation which maps the points z = 0, i, 2i into the points w = 5i , ∞, $\frac{i}{3}$.

12. a) Find the analytic function F(z) = u + iv if $u + v = \frac{x}{x^2+y^2}$ and F(1) = 1.

b) What is the image of the circle $|z| = c$ under the transformation $w = z + \frac{1}{z}$? Discuss the case when c = 1.

c) If F(z) is analytic, show that

$$\nabla^2 |F(z)|^2 = 4|F'(z)|^2$$

Module – II

13. a) Evaluate $\int_c |z|^2 \, dz$ where c is the rectangle with vertices $z = 0$, $z = 1$, $z = 1 + i$ and $z = i$.

b) Obtain the Laurent's series expansion of the function $\frac{1}{z - z^3}$ in the region $1 < |z + 1| < 2$.

c) Evaluate $\int_c \frac{z-2}{z(z-1)} dz$ where c is the circle $|z| = 2$ using Residue theorem.

14. a) Show that $\int_0^{2\pi} \frac{d\theta}{1+k\cos\theta} = \frac{2\pi}{\sqrt{1-k^2}} \, (k^2 < 1)$.

b) Evaluate $\int_0^{\infty} \frac{dx}{x^4 + 1}$.

Module – III

15. a) Find the root of $xe^x - 2 = 0$ which lies between 0 and 1 to four decimal places using the method of false position.

b) Using Gauss Seidal method solve the following system of equations.

$w - x + 3y - 3z = 3$

$2w + 3x + y - 11z = 1$

$5w - 2x + 5y - 4z = 5$

$3w + 4x - 7y + 2z = -7$

c) Estimate the value of F(22) from the following table

x :	20	25	30	35	40	45
F(x) :	354	332	291	260	231	204

16. a) The following table gives the velocity 'v' of a particle at time 't'.

t (seconds)	0	2	4	6	8	10	12
v (metres/sec)	4	6	16	34	60	94	136

Using Simpson's method, find the distance moved by the particle in 12 seconds.

b) Find y(.2) and y(.3) by Taylor's series method if

$y' + y = x^2$, $y(0) = 1$

c) Solve $\frac{dy}{dx} = \frac{3x+y}{2}$, $y(0) = 1$ using Range-Kutta method of order 4 and find y (.2).

(3×20=60 Marks)